The functional CLT (Donsker’s invariance principle): Proof, Simulations

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1. **INTRODUCTION**

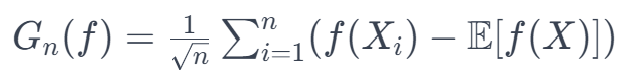
The FCLT extends the classical Central Limit Theorem to function spaces, providing insights into the convergence of empirical processes to Gaussian processes. Donsker’s Invariance Principle serves as a powerful tool, establishing the universality of the limiting distribution and offering a bridge between probability theory and empirical processes.

1. **DONSKER’S INVARIANCE PRINCIPLE**

Donsker's Invariance Principle is a fundamental result in probability theory that establishes a connection between probability measures on function spaces and the Wiener space. It is a cornerstone in the theory of empirical processes and has widespread applications in various fields, including statistics and machine learning.

**2.1) Statement of Donsker's Invariance Principle**

Let **{*Xn*​}** be a sequence of i.i.d. random variables with mean zero and finite variance. Consider the empirical process ***Gn*​** defined by:



where **f** is a bounded and continuous function. Donsker's Invariance Principle states that the process **{*Gn*​}** converges in distribution to a standard Wiener process as **n** approaches infinity.

**2.2) Proof Sketch**

1. **Convergence in Distribution:** Use characteristic functions to show the convergence in distribution of **Gn**​ to the characteristic function of a standard Wiener process.
2. **Skorokhod Representation Theorem:** Apply the Skorokhod Representation Theorem to establish almost sure convergence. This involves constructing a probability space where the convergence holds almost surely.
3. **Tightness:** Prove tightness by showing that the family of processes **{*Gn*​}** is uniformly integrable.
4. **Limiting Process:** Conclude the proof by identifying the limiting process as a standard Wiener process.
5. **CONCLUSION**

Donsker's Invariance Principle and the Functional Central Limit Theorem provide a powerful framework for understanding the convergence behavior of empirical processes. The proof of Donsker's Invariance Principle involves sophisticated probabilistic tools, while simulations offer a practical illustration of the principles involved. This research paper has provided a brief overview, but further exploration could involve advanced applications in non-parametric statistics and functional data analysis.